OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4752

Concepts for Advanced Mathematics (C2)

Monday **23 MAY 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

Section A (36 marks)

1 Differentiate $x+\sqrt{x^3}$. [4]

2 The nth term of an arithmetic progression is $6+5n$. Find the sum of the first 20 terms. [4]

3 Given that $\sin\theta=\dfrac{\sqrt{3}}{4}$, find in surd form the possible values of $\cos\theta$. [3]

4 A curve has equation $y = x+\dfrac{1}{x}$.

Use calculus to show that the curve has a turning point at $x = 1$.

Show also that this point is a minimum. [5]

5 **(i)** Write down the value of $\log_5 5$. [1]

(ii) Find $\log_3\left(\frac{1}{9}\right)$. [2]

(iii) Express $\log_a x+\log_a(x^5)$ as a multiple of $\log_a x$. [2]

6 Sketch the graph of $y = 2^x$.

Solve the equation $2^x = 50$, giving your answer correct to 2 decimal places. [5]

7 The gradient of a curve is given by $\dfrac{dy}{dx} = \dfrac{6}{x^3}$. The curve passes through $(1, 4)$.

Find the equation of the curve. [5]

8 **(i)** Solve the equation $\cos x = 0.4$ for $0° \leqslant x \leqslant 360°$.

(ii) Describe the transformation which maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. [5]

Section B (36 marks)

9

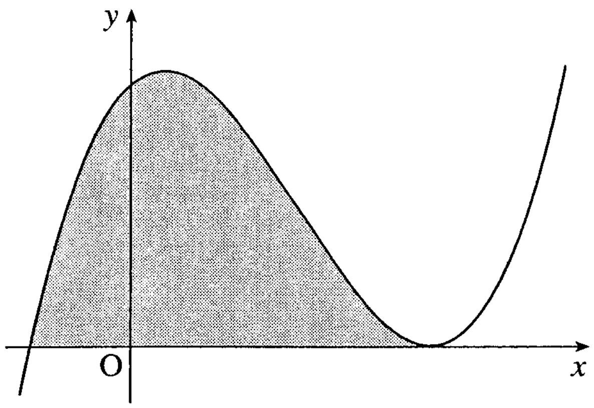


Fig. 9

Fig. 9 shows a sketch of the graph of $y = x^3 - 10x^2 + 12x + 72$.

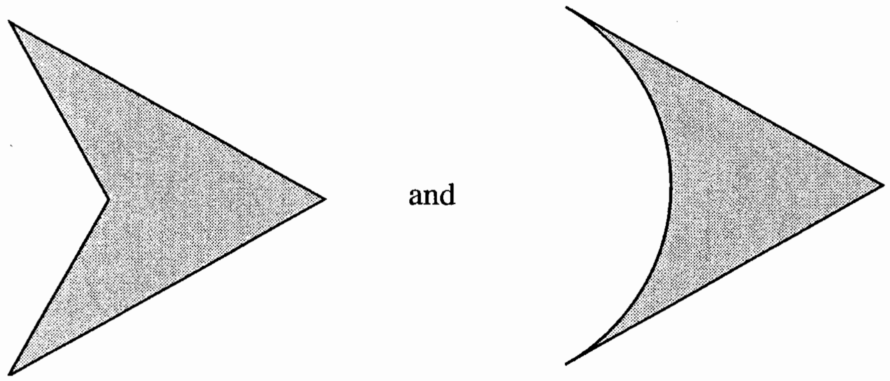
(i) Write down $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Find the equation of the tangent to the curve at the point on the curve where $x = 2$. [4]

(iii) Show that the curve crosses the x-axis at $x = -2$. Show also that the curve touches the x-axis at $x = 6$. [3]

(iv) Find the area of the finite region bounded by the curve and the x-axis, shown shaded in Fig. 9. [4]

10 *Arrowline Enterprises* is considering two possible logos:



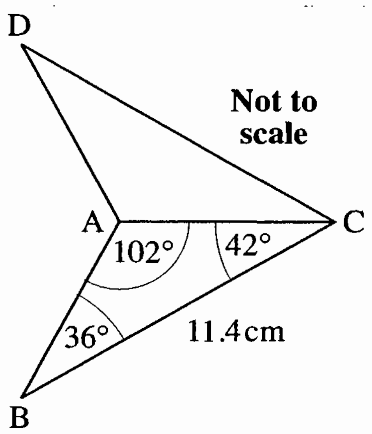


Fig. 10.1

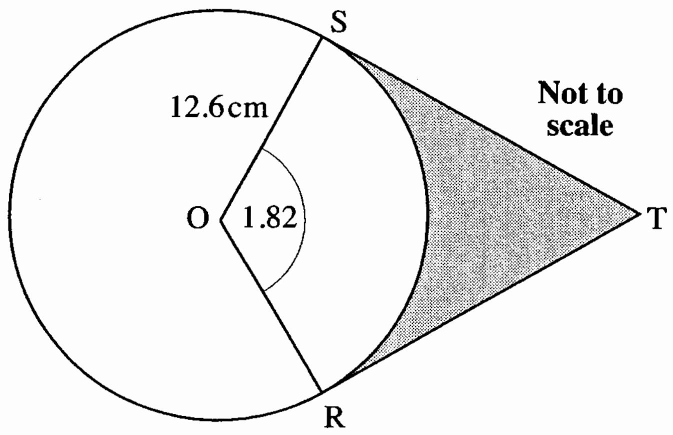


Fig. 10.2

(i) Fig. 10.1 shows the first logo ABCD. It is symmetrical about AC.

Find the length of AB and hence find the area of this logo. [4]

(ii) Fig. 10.2 shows a circle with centre O and radius 12.6 cm. ST and RT are tangents to the circle and angle SOR is 1.82 radians. The shaded region shows the second logo.

Show that ST $=$ 16.2 cm to 3 significant figures.

Find the area and perimeter of this logo. [8]

11 There is a flowerhead at the end of each stem of an oleander plant. The next year, each flowerhead is replaced by three stems and flowerheads, as shown in Fig. 11.

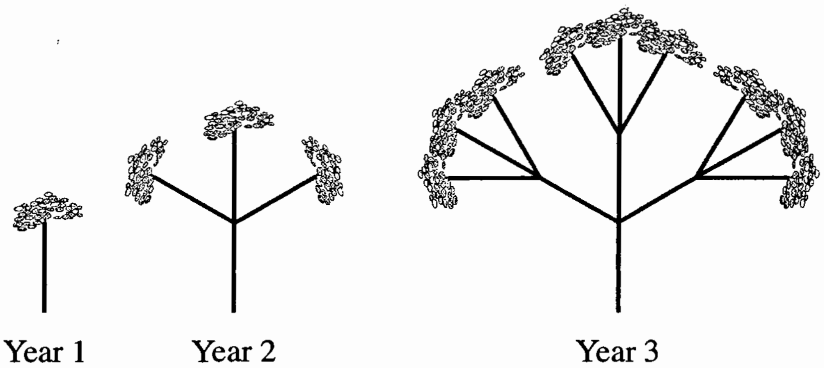


Fig. 11

(i) How many flowerheads are there in year 5? [1]

(ii) How many flowerheads are there in year n? [1]

(iii) As shown in Fig. 11, the total number of stems in year 2 is 4, (that is, 1 old one and 3 new ones). Similarly, the total number of stems in year 3 is 13, (that is, $1 + 3 + 9$).

Show that the total number of stems in year n is given by $\frac{3^n - 1}{2}$. [2]

(iv) Kitty's oleander has a total of 364 stems. Find

(A) its age, [2]

(B) how many flowerheads it has. [1]

(v) Abdul's oleander has over 900 flowerheads.

Show that its age, y years, satisfies the inequality $y > \frac{\log_{10}900}{\log_{10}3} + 1$.

Find the smallest integer value of y for which this is true. [4]

Mark Scheme 4752
June 2005

Section A

1	$1+\frac{3}{2}x^{\frac{1}{2}}$	1+3	B2 for $kx^{\frac{1}{2}}$, or M1 for $x^{\frac{3}{2}}$ seen before differentiation or B1 ft their $x^{\frac{3}{2}}$ correctly differentiated	4
2	1170	4	B1 for $a = 11$ and B1 for $d = 5$ or 20th term = 106 and M1 for 20/2[their (a) + their(106)] or 20/2[2their (a)+ (20-1)×their(d)] <u>OR</u> M1 for 6×20 and M2 for $5\left(\frac{20}{2}[20+1]\right)$ o.e.	4
3	±√13/4	3	B2 for (-) √13/4 or $\pm\sqrt{\frac{13}{16}}$ or M1 for √13 or $\sin^2\theta + \cos^2\theta = 1$ used	3
4	$x + x^{-1}$ soi $y' = 1 - 1/x^2$ subs $x = 1$ to get $y' = 0$ $y''=2x^{-3}$ attempted Stating $y'' > 0$ so min cao	B1 B1 B1 M1ft A1	 $1 - x^{-2}$ is acceptable Or solving $1 - x^{-2} = 0$ to obtain $x = 1$ or checking y' before and after $x = 1$ Valid conclusion First quadrant sketch scores B2	5
5	(i) 1 (ii) −2 (iii) 6log x	1 2 2	 M1 for $1/9=3^{-2}$ or $\log(1) - \log(3^2)$ base not requd; M1 for 5 log x or $\log(x^6)$	5
6	Correct curve thro' y axis (0, 1) indicated on sketch or table 5.64	G1 G1 3	y, y' & y'' all positive independent B2 for other versions of 5.64(3....) or B1 for other ans 5.6 to 5.7 or M1 for x log 2 = log 50 and M1 for x = log 50 ÷ log 2	5
7	$y = 7 - 3/x^2$ oe	5	B3 for (y =) $-3/x^2 + c$ [B1 for each of k/x^2, $k = -6/2$ and $+c$] and M1 for substituting (1, 4) in their attempted integration with + c, the constant of integration	5
8	(i) 66° or 66.4 or 66.5…. 293.58 …. to 3 or more sf cao (ii) stretch (one way) parallel to the x-axis sf 0.5	B1 B1 1 1 1	Allow 1.16 or 73.8 Lost for extras in range. Ignore extras outside the range Horizontal, from y axis, in x axis, oe	5
				36

Section B

9	**i**	$3x^2 - 20x + 12$	2	B1 if one error "+c" is an error	2
	ii	$y - 64 = -16(x - 2)$ o.e. eg $y = -16x + 96$	4	M1 for subst $x = 2$ in their y' A1 for $y' = -16$ and B1 for $y = 64$	4
	iii	Factorising $f(x) \equiv (x+2)(x-6)^2$ OR Expanding $(x+2)(x-6)^2$	B3 M2 E1	or B1 for $f(-2) = -8-40-24+72 = 0$ and B1 for $f'(6) = 0$ and B1dep for $f(6)=0$	3
	iv	$\frac{x^4}{4} - \frac{10x^3}{3} + 6x^2 + 72x$ value at ($x = 6$) ~ value at ($x = -2$) 341(.3..) cao	B2 M1 A1	-1 for each error Must have integrated $f(x)$	4
10	**i**	AB = 7.8(0), 7.798 to 7.799 seen area = 52.2 to 52.3	2 2	M1 for correct use of sine rule For long methods M1A1 for art 7.8 M1 for [2×][0.5 ×] their AB × 11.4 × sin 36°	4
	ii	tan 0.91 = ST/12.6 ST = 12.6 × tan 0.91 and completion (16.208...) area OSTR = [2×][0.5 ×]12.6 × their(16.2) nb 204. …. area of sector = $0.5 \times 12.6^2 \times 1.82$ =144.47… Logo = 59.6 to 60.0 arc = 12.6 × 1.82 [=22.9...] perimeter = 55.3 to 55.4	M1 E1 M1 M1 A1 A1 M1 A1	Accept 16.2 if ST is explicit but for long methods with pa check that their explicit expression = 16.2 oe using degrees soi by correct ans Accept 144, 144.5 oe using degrees	8
11	**i**	81	1		1
	ii	$(1\text{x})3^{n-1}$	1		1
	iii	(GP with) $a = 1$ and $r = 3$ clear correct use GP sum formula	M1 M1	or M1 for $= 1+3+9+ \ldots +3^{n-1}$	2
	iv	(A) 6 www (B) 243	2 1	M1 for $364 = (3^n - 1)/2$	3
	v	their (ii) > 900 $(y - 1)\log 3 > \log 900$ $y - 1 > \log 900 \div \log 3$ $y = 8$ cao	M1ft M1ft M1 B1	-1 once for = or < seen: condone wrong letter / missing brackets / no base	4

4752 - Concepts for Advanced Mathematics

General Comments

There were the usual number of excellent scripts produced by candidates with good grounding in arithmetic, algebra and calculus. Others showed serious weaknesses in these basic skills; $12 - 40 + 12 = 40$ or -40, $3^n - 1 = 728$ therefore $3^n = 727$, derivative of $x = 0$. Good candidates set out their work in a logical fashion and were well prepared for all topics covered, many scoring over 90%. A small fraction of the entry was totally unprepared and scored less than 10%. In Q.10 all parts can be solved quite simply if the appropriate moves are picked out; time could be well spent planning the work rather than spotting a side or angle that can be found leading to a long chain of operations culminating, possibly, in the right answer.

Comments on Individual Questions

1) Some had difficulty converting the second term to a power of x, so this question did not yield many marks to weaker candidates. The derivative of x was often wrong. A small minority attempted to integrate.

2) The question states that $6 + 5n$ is an A.P. so it would have been sensible to put n = 1,2,3 just to have a look at it in its simple form. A great many took the first term to be 6 and managed to score half marks, but many were confused and scored 0 or 1.

3) Not by design, but inevitably each session, candidates are given the opportunity to make some variations on the mistake $(a + b)^2 = a^2 + b^2$. In this question, the very large number who used $\sin^2 \theta + \cos^2\theta = 1$ could not resist taking the square root of all three terms. Some did it immediately with $\sin\theta + \cos\theta = 1$, not even bothering to write the correct form first; some did it later, even as late as $\cos^2\theta = 1 - 3/16$. It is puzzling that the arithmetic on the right-hand side was found so daunting. Those who put the information onto a right-angled triangle still had trouble squaring 4 and $\sqrt{3}$ and subtracting the results. The successful ones found $\sqrt{13}/4$. It was disappointing to see $\sqrt{13}/\sqrt{16}$ so often. Very few indeed gave both values of $\cos\theta$.

4) Many produced an excellent response here. Many had difficulty handling $x + x^{-1}$ and its derivative.

5) Some achieved full marks in fine style, others stumbled and fumbled. Expressing 1/9 as a power of 3 was found difficult. In part (iii) many failed to simplify $\log x + 5\log x$ to $6\log x$.

6) Many produced good looking sketch graphs, no doubt aided by their calculators. Marks were lost by cutting short the curve at the y-axis so that nothing appeared in the second quadrant. For full marks it was necessary to indicate in some way that the y intercept was 1.

$2^x = 50$ was solved by most, usually by using $x\log2 = \log50$ or by trial and improvement; this latter method often left them with insufficient accuracy.

7) Many weaker candidates thought that dy/dx was a gradient that could be used as m in y = mx + c, a fundamental misunderstanding. Others had difficulty integrating $6/x^3$. Many coped well and scored full marks.

8) (i) Nearly all produced the first quadrant solution and many went on to give the correct second solution.

(ii) The name "stretch" was essential here, together with the direction and stretch factor; and many gave all this information very briefly for 3 marks. Many unfortunate candidates did not know of the existence of a stretch and they wasted a lot of time sketching both graphs and describing in great detail the relationships between the two shapes discussing amplitudes, periods, squashes, oscillations and so on.

9) This yielded good marks, even for the poorer candidates. Most could differentiate the cubic and use their result to find the required tangent. Common errors were arithmetical and many could not resist converting their gradient of -16 to 1/16 before using their procedure for a straight line. In part (iii) some recognised that the factors $(x + 2)(x - 6)^2$ were important and they scored full marks if they expanded them correctly to fit the cubic or conversely, used a valid method to obtain them from the cubic. Those who did not could score marks for showing f(-2) = 0 of for $f^1(6) = 0$. In part (iv) the method was generally well known but some integrated their derivative, some simply used the cubic as their integrand and some differentiated the cubic before evaluating using their limits. Although the arithmetic was quite involved most good candidates got it right, weaker candidates got it wrong.

10) (i) The sine rule and the formula A = ½ ab sin C were well known and most scored 3 of the 4 marks; many forgot to double the area of triangle ABC to get the area of the logo. Some dropped a perpendicular from A to BC and if they proceeded correctly they achieved the marks; if they assumed that line bisected BC, and quite a few did, they did not score. A few joined DB and produced CA and went through a long chain of calculations finding all angles, sides and areas.

(ii) Those who used right-angled triangle OST could score 4 of the 8 marks immediately. The work with the sector and arc, which they did not find difficult, gave them the other 4 marks quite quickly. Alas, SR and OT were joined and again many angles, sides and areas were found, with maybe, the side and the area needed.

11) (i)&(ii) Most candidates recognized the progression 1, 3, 9 and its continuation 27, 81 and so even the weaker ones found 81. Many also realised it was a GP and gave the n^{th} term $ar^{n-1} = 3^{n-1}$.

(iii) Although the question gives a broad hint that the number of stems in year n is the sum of the above G.P. to that year, many did not find it obvious and substituting a = 1, r = 3 into $a(r^n – 1)/r – 1)$ was not always seen. Many simply tested the printed expression for n = 1,2,3.

(iv) Most wrote down $364 = (3^n – 1)/2$ and many got to $3^n = 729$. This was solved using logs or by trialling and the answer 6 was often achieved. Having got year 6, it was no problem to get the number of flowerheads.

(v) It is generally better to solve inequalities as such rather than as equations with an appropriate symbol inserted in the final line, so there was a penalty of 1 mark for those who used a wrong symbol anywhere. Those who began $3^{y-1} > 900$ very often proceeded with the next three moves correctly and scored 3 marks. It was surprising that not all who got to $y > 7.19$ converted to 8.